HHHS MATH DEPARTMENT

AP CALCULUS

SUMMER PREP PACKET

Welcome to AP Calculus! This summer review packet includes prerequisite topics from algebra, trigonometry, and geometry that are necessary for success in calculus. You will be given an opportunity to ask questions on these topics in September.

Instructions:

- Complete the entire packet neatly on loose-leaf (or unlined paper), showing all work. Clearly number each problem.
- Check your answers and retry problems until you have mastered the contents of this packet.
- This will be collected the first day of school for a weighted homework grade. You will also be assessed on your mastery of these topics in the second week of school (following the first Wednesday after-school extra-help session).

Contents:

- *108 Practice Questions and Answers*
- *29 Additional Trigonometry Practice Problems*
  [You must be able to do these without having to draw (or refer to) a full unit circle. You should be able to answer them in your head with possibly a rough sketch of the particular radian measure.]
- *Worked-out solutions to the problems with review tips.*

In addition to the worked-out solutions, the following are helpful resources if you need assistance with a topic.
https://www.khanacademy.org/
http://www.wyzant.com/resources/lessons/math/precalculus

Or google search each topic as you have trouble.

My email: cassanop@hhschools.org

AP Calculus AB Summer Review Packet

Simplify

1. $\dfrac{x^3-9x}{x^2-7x+12}$

2. $\dfrac{x^2-2x-8}{x^3+x^2-2x}$

3. $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

4. $\dfrac{9-x^{-2}}{3-x^{-1}}$

Rationalize the denominator

5. $\dfrac{2}{\sqrt{3}+\sqrt{2}}$

6. $\dfrac{4}{1-\sqrt{5}}$

7. $\dfrac{1-\sqrt{5}}{1+\sqrt{3}}$

Write each of the following expressions in the form of $ca^p b^q$ where $c, p,$ and q are numbers

8. $\dfrac{(2a^2)^3}{b}$

9. $\sqrt{9ab^3}$

10. $\dfrac{a(2/b)}{3/a}$

11. $\dfrac{ab-a}{b^2-b}$

12. $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$

13. $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2 \left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

Solve for x. Do not use a calculator

14. $5^{(x+1)} = 25$

15. $\frac{1}{3} = 3^{2x+2}$

16. $\log_2 x = 3$

17. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Simplify

18. $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

19. $2\log_4 9 - \log_2 3$

20. $3^{2\log_3 5}$

Simplify

21. $\log_{10} 10^{1/2}$

22. $\log_{10} \frac{1}{10^x}$

23. $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$

Solve the following equations for the indicated variable

24. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, for a

25. $V=2(ab+bc+ca)$, for a

26. $A=2\pi r^2+2\pi rh$, for positive h

27. $A=P+\pi rP$, for P

28. $2x-2yd=y+xd$, for d

29. $\frac{2x}{4\pi}+\frac{1-x}{2}=0$, for x

For each equation complete the square and reduce to one of the standard forms $y-y_1=A(x-x_1)^2$ or $x-x_1=(y-y_1)^2$

30. $y=x^2+4x+3$

31. $3x^2+3x+2y=0$

32. $9y^2-6y-9-x=0$

Factor completely

33. x^6-16x^4

34. $4x^3-8x^2-25x+50$

35. $8x^3+27$

36. x^4-1

Find all real solutions

37. $x^6-16x^4=0$

38. $4x^3-8x^2-25x+50=0$

39. $8x^3+27=0$

Solve for x

40. $3sin^2x=cos^2x; \quad 0\le x<2\pi$

41. $cos^2x-sin^2x=\sin x; -\pi<x\le\pi$

42. $\tan x+\sec x=2\cos x; -\infty<x<\infty$

Without using a calculator, evaluate the following:

43. $\cos 210°$

44. $\sin\frac{5\pi}{4}$

45. $\tan^{-1}(-1)$

46. $\sin^{-1}(-1)$

47. $\cos\frac{9\pi}{4}$

48. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

49. $\tan\left(\frac{7\pi}{6}\right)$

50. $\cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

Given the graph of $y = \sin x$, sketch the graphs of:

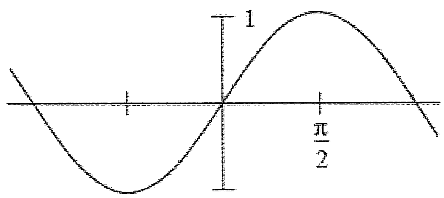
51. $\sin\left(x - \frac{\pi}{4}\right)$

52. $\sin\left(\frac{\pi}{2}x\right)$

53. $2 \sin x$

54. $\cos x$

55. $\frac{1}{\sin x}$



Solve the equations

56. $4x^2 + 12x + 3 = 0$

57. $2x + 1 = \frac{5}{x+2}$

58. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Find the remainders on division of

59. $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$

60. $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

61. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

62. Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$ (all solutions are rational and between ± 1)

Solve the inequalities. Give the solution in interval notation

63. $x^2 + 2x - 3 \leq 0$

64. $\frac{2x-1}{3x-2} \leq 1$

65. $\frac{2}{2x+3} > \frac{2}{x-5}$

Solve for x. Give the solution in interval notation

66. $|-x + 4| \leq 1$

67. $|5x - 2| = 8$

68. $|2x + 1| > 3$

Determine the equation of the following lines

69. The line through $(-1, 3)$ and $(2, -4)$

70. The line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$

71. The line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$

72. Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$

73. Shade the region in the xy-plane that is described by the inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$

Find the equations of the following circles:

74. The circle with center at $(1, 2)$ that passes through the point $(-2, -1)$

75. The circle that passes through the origin and has intercepts equal to 1 and 2 on the x and y axes respectively.

76. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$ find the center and the radius

77. Find the domain of $\frac{3x+1}{\sqrt{x^2+x-2}}$

Find the domain and range of:

78. $f(x) = 7$

79. $g(x) = \frac{5x-3}{2x+1}$

80. $f(x) = \frac{|x|}{x}$

Simplify $\frac{f(x+h)-f(x)}{h}$ when

81. $f(x) = 2x + 3$

82. $f(x) = \frac{1}{x+1}$

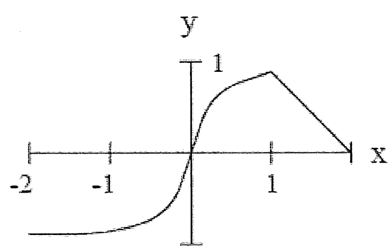
83. $f(x) = 3x^2 - x + 5$

The graph of the functions $y = f(x)$ is given as follows: Determine the graphs of the functions:

84. $f(x + 1)$

85. $f(-x)$

86. $|f(x)|$



Sketch the graphs of the functions

87. $g(x) = |3x + 2|$

88. $h(x) = |x(x - 1)|$

89. The graph of a quadratic function has x-intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.

90. Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$

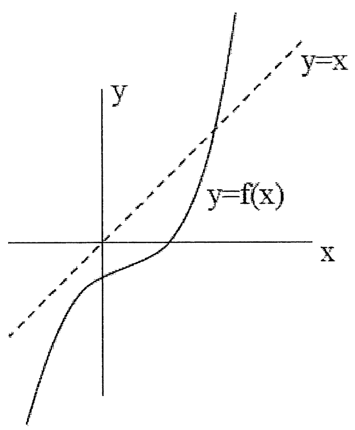
Find the inverse of the functions

91. $f(x) = 2x + 3$

92. $f(x) = \frac{x+2}{5x-1}$

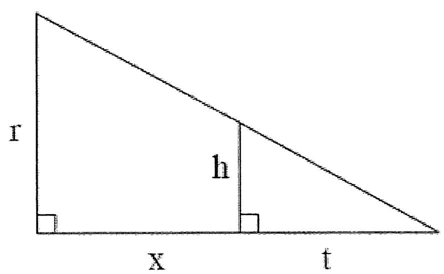
93. $f(x) = x^2 - 2x - 1, x > 0$

94. A function $f(x)$ has the graph below. Sketch the graph of the inverse function $f^{-1}(x)$.

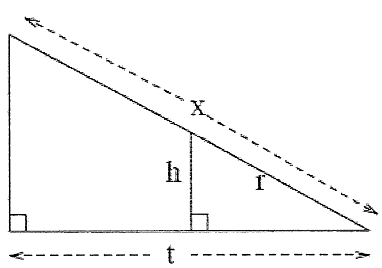


For problems 96 and 97, express x in terms of the other variables in the picture:

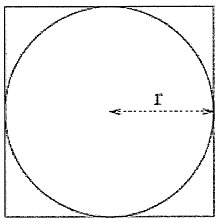
95.



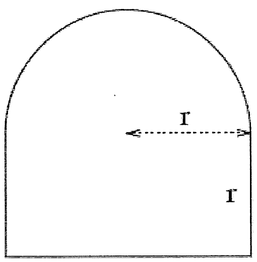
96.



97. Find the ration of the area inside the square but outside the circle to the area of the square in the picture below



98. Find the formula for the perimeter of the window of the shape in the picture below



99. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is $10m$ high and has a radius of $3\ m$ as the top. If the water is $5\ m$ deep (in the middle) what is the surface area of the top of the water?

100. Two cars start moving from the same point. One travels south at $100\ km/hr$, the other west at $50\ km/hr$. How far apart are they two hours later?

101. A kite is $100\ m$ above the ground. If there are $200\ m$ of string out, what is the angle between the sting and the horizontal. (Assume that the string is perfectly straight.)

If $f(x) = 2x - 3$ and $g(x) = \sqrt{3x - 1}$, Find:

102. $f(g(x))$

103. $g(f(x))$

104. If $f(x) = \frac{3}{x}$ and $g(x) = \frac{x}{2x-1}$, Find $f(g(x))$ and state its domain.

Decompose each composition function into individual function. (If $y = f(u)$, identify $u$and rewrite y in terms of u)

105. $y = \sin 3x$

106. $y = \sqrt[5]{2x + 1}$

107. $y = (x^2 - 2x + 5)^5$

108. $y = cos^2 x$

Name: ________________________ **Class:** ___________________ **Date:** __________ **ID: A**

AP Calculus Trig Practice (For Summer Packet) NO CALCULATOR

1. **INSTRUCTIONS: Answer each of the following without a calculator taking no longer than 20 seconds per question. Answers must be exact (no decimals) and radicals need to be simplified/rationalized.**

   $\sin \frac{2\pi}{3} =$

2. $\sec \frac{\pi}{2} =$

3. $\cos \pi =$

4. $\csc \frac{7\pi}{6} =$

5. $\cot 0 =$

6. $\sin \frac{11\pi}{6} =$

7. $\tan \frac{-3\pi}{4} =$

8. $\cos \frac{7\pi}{3} =$

9. $\cot \frac{\pi}{2} =$

10. $\sin \frac{-3\pi}{2} =$

11. $\sin \frac{19\pi}{6} =$

12. Evaluate (if possible) the sine, cosine, and tangent of the real number.

    $t = -\frac{3\pi}{4}$

    sin t = cos t = tan t =

13. Evaluate (if possible) the sine, cosine, and tangent of the real number.

    $t = \frac{-101\pi}{6}$

    sin t = cos t = tan t =

14. Evaluate (if possible) the sine, cosine, and tangent of the real number.

    $t = \frac{205\pi}{2}$

    sin t = cos t = tan t =

15. Evaluate (if possible) the sine, cosine, and tangent of the real number.

    $t = 150°$

    sin t = cos t = tan t =

16. Evaluate (if possible) the sine, cosine, and tangent of the real number.

$t = -60°$

sin t = cos t = tan t =

17. Evaluate (if possible) the secant, cotangent, and cosecant of the real number.

$t = \frac{4\pi}{3}$

sec t = cot t = csc t =

18. Evaluate (if possible) the secant, cotangent, and cosecant of the real number.

$t = -3\pi$

sec t = cot t = csc t =

19. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\tan\theta = \sqrt{3}$

20. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\sin\theta = -\frac{\sqrt{3}}{2}$

21. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\csc\theta = 2$

22. Evaluate $\arccos\frac{\sqrt{3}}{2}$ without using a calculator.

23. Evaluate $\arcsin\frac{\sqrt{2}}{2}$ without using a calculator.

24. Evaluate $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ without using a calculator.

25. Evaluate $\sin^{-1}(0)$ without using a calculator.

26. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\cos\theta = \frac{\sqrt{2}}{2}$

27. Find ALL (using correct notation) values of θ for which the following is true $\sin 2\theta = -\frac{1}{2}$

.

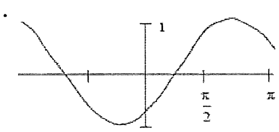
28. Find ALL (using correct notation) values of θ for which the following is true $\cos 6\theta = -1$

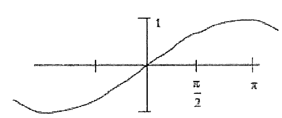
.

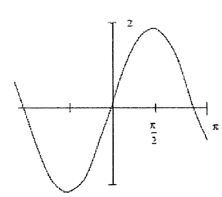
29. Find ALL (using correct notation) values of θ for which the following is true

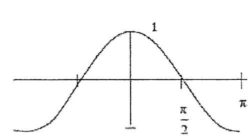
$\tan\frac{\theta}{5} = \textit{undefined}$

Answers

1. $\frac{x^2+3x}{x-4}$
2. $\frac{x-4}{x^2-x}$
3. $\frac{5x}{x+5}$
4. $\frac{3x+1}{x}$
5. $2(\sqrt{3}-\sqrt{2})$
6. $-1-\sqrt{5}$
7. $\frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{-2}$
8. $8a^6b^{-1}$
9. $3a^{1/2}b^{3/2}$
10. $\frac{2}{3}a^2b^{-1}$
11. ab^{-1}
12. $a^{-3/2}b$
13. $a^{5/6}b^{1/2}$
14. 1
15. $-\frac{3}{2}$
16. 8
17. $\pm\frac{4}{25}$
18. $\log_2(5(x+1))$
19. $\log_2 3$
20. 25
21. $\frac{1}{2}$
22. $-x$
23. $2\log_{10} x$
24. $\frac{bcx}{bc-cy-bz}$
25. $\frac{V-2bc}{2(b+c)}$
26. $\frac{A-2\pi r^2}{2\pi r}$
27. $\frac{A}{1+\pi r}$
28. $\frac{2x-y}{x+2y}$
29. $\frac{\pi}{\pi-1}$
30. $y+1=(x+2)^2$
31. $y-\frac{3}{8}=-\frac{3}{2}\left(x+\frac{1}{2}\right)^2$
32. $x+10=9\left(y-\frac{1}{3}\right)^2$
33. $x^4(x-4)(x+4)$
34. $(x-2)(2x-5)(2x+5)$
35. $(2x+3)(4x^2-6x+9)$
36. $(x-1)(x+1)(x^2+1)$
37. $0, \pm 4$
38. $2, \pm\frac{5}{2}$
39. $-\frac{3}{2}$
40. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
41. $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
42. $\frac{\pi}{6}+2k\pi$ and $\frac{5\pi}{6}+2k\pi$ where $k \epsilon I$
43. $-\frac{\sqrt{3}}{2}$
44. $-\frac{\sqrt{2}}{2}$
45. $-\frac{\pi}{4}$
46. $-\frac{\pi}{2}$
47. $\frac{\sqrt{2}}{2}$
48. $\frac{\pi}{3}$
49. $\frac{\sqrt{3}}{3}$
50. $\frac{3\pi}{4}$
51. 

52. 

53. 

54. 

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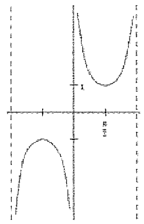


56. $\frac{-3\pm\sqrt{6}}{2}$

57. $\frac{1}{2}$ or -3

58. $-\frac{1}{2}$

59. -89

60. x^2+3

61. $-\frac{1}{3}$ or $\frac{1}{4}$

62. $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{2}$

63. $[-3,1]$

64. $\left(-\infty,\frac{2}{3}\right)\cup[1,\infty)$

65. $(-\infty,-8)\cup\left(-\frac{3}{2},5\right)$

66. $[3,5]$

67. 2 and $-\frac{6}{5}$

68. $(-\infty,-2)\cup(1,\infty)$

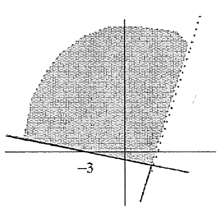
69. $7x+3y=2$

70. $3x+2y=1$

71. $y=3$

72. $(2,-1)$

73.



74. $(x-1)^2+(y-2)^2=18$

75. $\left(x-\frac{1}{2}\right)^2+(y-1)^2=\frac{5}{4}$

76. Center $=(-3,2)$, radius $=\sqrt{10}$

77. $(-\infty,-2)\cup(1,\infty)$

78. Domain $(-\infty,\infty)$ Range $\{7\}$

79. Domain $\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$

Range $\left(-\infty,\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

80. Domain $(-\infty,0)\cup(0,\infty)$

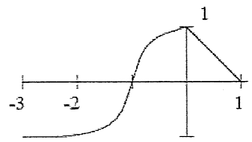
Range $\{-1,1\}$

81. 2

82. $\frac{-1}{(x+1)(x+h+1)}$

83. $6x+3h-1$

84.



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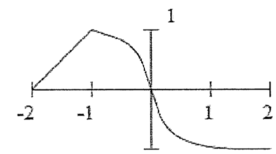

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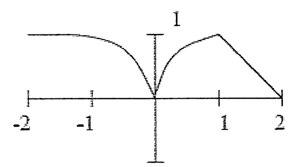

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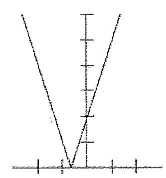

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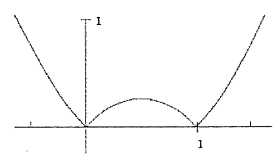
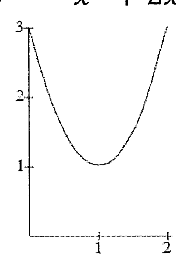


89. $y=-x^2+2x+3$

90.

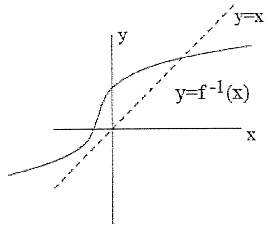


91. $f^{-1}=\frac{x-3}{2}$

92. $f^{-1}=\frac{x+2}{5x-1}$

93. $f^{-1} = 1 + \sqrt{x+2}$ for $x > -1$

94.



95. $x = t\left(\frac{r-h}{h}\right)$

96. $x = \frac{rt}{\sqrt{r^2-h^2}}$

97. $1 - \frac{\pi}{4}$

98. $4r + \pi r$

99. $\frac{9\pi}{4}$

100. $100\sqrt{5}\ KM$

101. $\frac{\pi}{6}$

102. $2\sqrt{3x-1} - 3$

103. $\sqrt{6x-10}$

104. $\frac{6x-3}{x}$

Domain $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

105. Let $u = 3x$, then $y = \sin u$

106. Let $u = 2x + 1$, then $y = \sqrt[5]{u}$

107. Let $u = x^2 - 2x + 5$,
then $y = u^5$

108. Let $u = \cos x$, then $y = u^2$

Note: You should not have to draw an entire unit circle to answers these.

Name: Key Class: ________________ Date: __________ ID: A

AP Calculus **Trig Practice (For Summer Packet)** **NO CALCULATOR**

1. **INSTRUCTIONS: Answer each of the following without a calculator taking no longer than 20 seconds per question. Answers must be exact (no decimals) and radicals need to be simplified/rationalized.**

   $\sin\frac{2\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$

2. $\sec\frac{\pi}{2} = \frac{1}{\cos\frac{\pi}{2}} = \frac{1}{0} = \boxed{\text{undefined}}$

3. $\cos\pi = \boxed{-1}$

4. $\csc\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}} = \frac{1}{-1/2} = \boxed{-2}$

5. $\cot 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0} = \boxed{\text{undefined}}$

6. $\sin\frac{11\pi}{6} = \boxed{-\frac{1}{2}}$

7. $\tan\frac{-3\pi}{4} = \tan\frac{5\pi}{4} = \boxed{1}$

8. $\cos\frac{7\pi}{3} = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

9. $\cot\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\pi/2} = \frac{0}{1} = \boxed{0}$

10. $\sin\frac{-3\pi}{2} = \sin\frac{\pi}{2} = \boxed{1}$

11. $\sin\frac{19\pi}{6} = \sin\frac{7\pi}{6} = \boxed{-\frac{1}{2}}$

12. Evaluate (if possible) the sine, cosine, and tangent of the real number.

    $t = -\frac{3\pi}{4}$

    sin t = $\boxed{\frac{-\sqrt{2}}{2}}$ cos t = $\boxed{\frac{-\sqrt{2}}{2}}$ tan t = $\boxed{1}$

13. Evaluate (if possible) the sine, cosine, and tangent of the real number. *

    $t = \frac{-101\pi}{6} + \left(\frac{12\pi}{6}\right)9 = \frac{-101\pi}{6} + \frac{108\pi}{6} = \frac{7\pi}{6}$

    sin t = $\boxed{-\frac{1}{2}}$ cos t = $\boxed{-\sqrt{3}/2}$ tan t = $\boxed{\frac{\sqrt{3}}{3}}$

14. Evaluate (if possible) the sine, cosine, and tangent of the real number. *

    $t = \frac{205\pi}{2} - \left(\frac{4\pi}{2}\right)(51) = \frac{205\pi}{2} - \frac{204\pi}{2} = \frac{\pi}{2}$

    sin t = $\boxed{1}$ cos t = $\boxed{0}$ tan t = $\boxed{\text{und.}}$

15. Evaluate (if possible) the sine, cosine, and tangent of the real number.

    $t = 150° = \frac{150}{1} \cdot \frac{\pi}{180} \text{ rad} = \frac{5\pi}{6} \text{ rad}$

    sin t = $\boxed{\frac{1}{2}}$ cos t = $\boxed{\frac{-\sqrt{3}}{2}}$ tan t = $\boxed{\frac{-\sqrt{3}}{3}}$

*Remember: You can add multiples of 2π to find coterminal points on the unit circle.

For 22 - 25: Remember inverse sine and inverse tangent have a range $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Inverse cosine $0 \le \theta \le \pi$

Name: Key

ID: A

16. Evaluate (if possible) the sine, cosine, and tangent of the real number.

$t = -60°$

sin t = $\boxed{-\frac{\sqrt{3}}{2}}$ cos t = $\boxed{\frac{1}{2}}$ tan t = $\boxed{-\sqrt{3}}$

17. Evaluate (if possible) the secant, cotangent, and cosecant of the real number.

$t = \frac{4\pi}{3}$

$\frac{1}{\cos\frac{4\pi}{3}}$ $\frac{1}{\tan\frac{4\pi}{3}}$ $\frac{1}{\sin\frac{4\pi}{3}}$

sec t = $\boxed{-2}$ cot t = $\boxed{\frac{\sqrt{3}}{3}}$ csc t = $\boxed{\frac{-2\sqrt{3}}{3}}$

18. Evaluate (if possible) the secant, cotangent, and cosecant of the real number.

$t = -3\pi = \pi$

sec t = $\boxed{-1}$ cot t = $\boxed{\text{Und.}}$ csc t = $\boxed{\text{und.}}$

19. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\tan \theta = \sqrt{3}$ $\boxed{\frac{\pi}{3}}$ & $\boxed{\frac{4\pi}{3}}$

20. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\sin \theta = -\frac{\sqrt{3}}{2}$ $\boxed{\frac{4\pi}{3}}$ & $\boxed{\frac{5\pi}{3}}$

21. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\csc \theta = 2$

$\sin\theta = \frac{1}{2}$ $\boxed{\frac{\pi}{6}}$ & $\boxed{\frac{5\pi}{6}}$

22. Evaluate $\arccos \frac{\sqrt{3}}{2}$ without using a calculator. $\boxed{\frac{\pi}{6}}$

23. Evaluate $\arcsin \frac{\sqrt{2}}{2}$ without using a calculator. $\boxed{\frac{\pi}{4}}$

24. Evaluate $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ without using a calculator. $\boxed{-\frac{\pi}{6}}$

25. Evaluate $\sin^{-1}(0)$ without using a calculator. $\boxed{0}$

26. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\cos \theta = \frac{\sqrt{2}}{2}$ $\boxed{\frac{\pi}{4}}$ & $\boxed{\frac{7\pi}{4}}$

27. Find ALL (using correct notation) values of θ
* for which the following is true $\sin 2\theta = -\frac{1}{2}$

$2\theta = \frac{11\pi}{6} + 2n\pi$ $2\theta = \frac{7\pi}{6} + 2n\pi$

$\boxed{\theta = \frac{11\pi}{12} + n\pi}$ $\boxed{\theta = \frac{7\pi}{12} + n\pi}$

28. Find ALL (using correct notation) values of θ
* for which the following is true $\cos 6\theta = -1$

$6\theta = \pi + 2n\pi$

$\boxed{\theta = \frac{\pi}{6} + \frac{n\pi}{3}}$

29. Find ALL (using correct notation) values of θ
* for which the following is true

$\tan \frac{\theta}{5} = \textit{undefined}$

$\frac{\theta}{5} = \frac{\pi}{2} + n\pi \rightarrow \boxed{\theta = \frac{5\pi}{2} + 5n\pi}$

* Where n is an integer.

AP Calculus Summer Packet

Worked-Out Solutions

1) $$\frac{x^3-9x}{x^2-7x+12} = \frac{x(x^2-9)}{(x-3)(x-4)} = \frac{x\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-4)}$$

$$= \boxed{\frac{x(x+3)}{x-4}}$$

Please make note if you think there are any errors in these solutions. We will discuss when we return to school in September.

-) $$\frac{x^2-2x-8}{x^3+x^2-2x}$$

$$= \frac{(x-4)(x+2)}{x(x^2+x-2)}$$

$$= \frac{(x-4)\cancel{(x+2)}}{x\cancel{(x+2)}(x-1)}$$

$$= \boxed{\frac{x-4}{x(x-1)}}$$

3) $$\frac{\left(\frac{1}{x}-\frac{1}{5}\right)(25x^2)}{\left(\frac{1}{x^2}-\frac{1}{25}\right)(25x^2)} = \frac{25x-5x^2}{25-x^2} = \frac{5x(\cancel{5-x})}{(\cancel{5-x})(5+x)}$$

$$= \boxed{\frac{5x}{5+x}}$$

4) $$\frac{9-x^{-2}}{3-x^{-1}} = \frac{\left(9-\frac{1}{x^2}\right)x^2}{\left(3-\frac{1}{x}\right)x^2} = \frac{9x^2-1}{3x^2-x} = \frac{(\cancel{3x-1})(3x+1)}{x(\cancel{3x-1})}$$

$$= \boxed{\frac{3x+1}{x}}$$

5) $$\frac{2}{\sqrt{3}+\sqrt{2}}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}-2\sqrt{2}}{3-2} = 2\sqrt{3}-2\sqrt{2} = \boxed{2(\sqrt{3}-\sqrt{2})}$$

6) $$\frac{4}{1-\sqrt{5}}\cdot\frac{1+\sqrt{5}}{1+\sqrt{5}} = \frac{4(1+\sqrt{5})}{1-5} = \frac{4(1+\sqrt{5})}{-4} = \boxed{-1-\sqrt{5}}$$

7) $$\frac{(1-\sqrt{5})}{(1+\sqrt{3})}\cdot\frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{1-3} = \boxed{\frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{-2}}$$

---

8) $$\frac{(2a^2)^3}{b} = \frac{8a^6}{b} = \boxed{8a^6b^{-1}}$$

9) $$\sqrt{9ab^3} = 9^{\frac{1}{2}}a^{\frac{1}{2}}(b^3)^{\frac{1}{2}} = \boxed{3a^{\frac{1}{2}}b^{\frac{3}{2}}}$$

10) $$\frac{a(2/b)}{3/a} = \frac{2a}{b}\div\frac{3}{a} = \frac{2a}{b}\cdot\frac{a}{3} = \boxed{\frac{2}{3}a^2b^{-1}}$$

11) $\frac{ab-a}{b^2-b} = \frac{a(b-1)}{b(b-1)} = \frac{a}{b} = \boxed{1\,a\,b^{-1}}$

12) $\frac{a^{-1}}{(b^{-1})\sqrt{a}} = \frac{a^{-1}}{b^{-1}a^{\frac{1}{2}}} = \boxed{a^{-\frac{3}{2}}\,b^{1}}$

13) $\left(\frac{a^{2/3}}{b^{\frac{1}{2}}}\right)^2\left(\frac{b^{3/2}}{a^{\frac{1}{2}}}\right) = \left(\frac{a^{4/3}}{b^{2/2}}\right)\left(\frac{b^{3/2}}{a^{\frac{1}{2}}}\right) = \boxed{a^{\frac{5}{6}}\,b^{\frac{1}{2}}}$

4) $5^{(x+1)} = 25$

$5^{(x+1)} = 5^2$

$x+1 = 2$

$\boxed{x = 1}$

15) $\frac{1}{3} = 3^{2x+2}$

$3^{-1} = 3^{2x+2}$

$-1 = 2x+2$

$-3 = 2x$

$\boxed{-\frac{3}{2} = x}$

16) $\log_2 x = 3$

$2^3 = x$

$\boxed{8 = x}$

17)

$\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

$\log_3 x^2 = \log_3 4^2 - \log_3 5^4$

$\log_3 x^2 = \log_3\left(\frac{4^2}{5^4}\right)$

$x^2 = \frac{4^2}{5^4}$

$\boxed{x = \frac{\pm 4}{25}}$

18) $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

$= \log_2\left(\frac{5(x^2-1)}{x-1}\right) = \log_2\left(\frac{5\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}\right)$

$= \boxed{\log_2(5(x+1))}$

19) $2\log_4 9 - \log_2 3$ ← use change of base formula

$= 2\left(\frac{\log_2 9}{\log_2 4}\right) - \log_2 3$

$\log_2 4 = 2$ →

b/c $2^2 = 4$

$= \log_2 9 - \log_2 3$

$= \log_2\left(\frac{9}{3}\right) = \boxed{\log_2 3}$

20) $3^{2\log_3 5} = 3^{\log_3 5^2} = 5^2 = \boxed{25}$

21) $\log_{10} 10^{\frac{1}{2}} = \boxed{\frac{1}{2}}$

22) $\log_{10}\left(\frac{1}{10^x}\right) = \log_{10} 10^{-x} = \boxed{-x}$

23) $2\log_{10}\sqrt{x} + 3\log_{10} x^{\frac{1}{3}}$

$\log_{10}(\sqrt{x})^2 + \log_{10}\left(x^{\frac{1}{3}}\right)^3$

$\log_{10} x + \log_{10} X$

$\log_{10} x^2 = \boxed{2\log_{10} X}$

24) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\frac{x}{a} = 1 - \frac{y}{b} - \frac{z}{c}$

$x = a\left(1 - \frac{y}{b} - \frac{z}{c}\right)$

$\frac{x}{1 - \frac{y}{b} - \frac{z}{c}} = a$

$\boxed{\frac{xbc}{bc - cy - bz} = a}$

25) $V = 2(ab + bc + ca)$

$\frac{V}{2} = ab + bc + ca$

$\frac{V}{2} - bc = ab + ca$

$\frac{V}{2} - bc = a(b+c)$

$\frac{V}{2(b+c)} - \frac{bc}{(b+c)} = a$

$\boxed{\frac{V - 2bc}{2(b+c)} = a}$

26) $A = 2\pi r^2 + 2\pi r h$

$A - 2\pi r^2 = 2\pi r h$

$\boxed{\frac{A - 2\pi r^2}{2\pi r} = h}$

27) $A = P + \pi r P$

$A = P(1 + \pi r)$

$\boxed{\frac{A}{1+\pi r} = P}$

28) $2x - 2yd = y + xd$

$2x - y = 2yd + xd$

$2x - y = d(2y + x)$

$\boxed{\frac{2x-y}{2y+x} = d}$

29) $4\pi\left(\frac{2x}{4\pi} + \frac{1-x}{2}\right) = (0)\,4\pi$

$2x + 2\pi - 2\pi x = 0$

$x + \pi - \pi x = 0$

$\pi = \pi x - x$

$\pi = x(\pi - 1)$

$\boxed{\frac{\pi}{\pi - 1} = x}$

30) $y = x^2 + 4x + 3$

$y = (x^2 + 4x + \underline{4}) + 3 - \underline{4}$

$y = (x+2)^2 - 1$

$\boxed{y + 1 = (x+2)^2}$

31) $3x^2 + 3x + 2y = 0$

$3x^2 + 3x = -2y$

$x^2 + x + \frac{1}{4} = -\frac{2}{3}y + \frac{1}{4}$

$\left(-\frac{3}{2}\right)\left(\left(x + \frac{1}{2}\right)^2\right) = \left(-\frac{2}{3}y + \frac{1}{4}\right)\left(-\frac{3}{2}\right)$

$\boxed{-\frac{3}{2}\left(x + \frac{1}{2}\right)^2 = y - \frac{3}{8}}$

32) $\frac{1}{9}(9y^2 - 6y - 9 - x) = (0)\,\frac{1}{9}$

$y^2 - \frac{2}{3}y - 1 - \frac{1}{9}x = 0$

$y^2 - \frac{2}{3}y = \frac{1}{9}x + 1$

$y^2 - \frac{2}{3}y + \frac{1}{9} = \frac{1}{9}x + 1 + \frac{1}{9}$

$9\left(\left(y - \frac{1}{3}\right)^2\right) = \left(\frac{1}{9}x + \frac{10}{9}\right)9$

$\boxed{9\left(y - \frac{1}{3}\right)^2 = x + 10}$

33) $x^6 - 16x^4$

$x^4(x^2 - 16)$

$\boxed{x^4(x-4)(x+4)}$

36) $x^4 - 1$

$(x^2 - 1)(x^2 + 1)$

$\boxed{(x-1)(x+1)(x^2+1)}$

37) $x^4(x-4)(x+4) = 0$

$\boxed{x = 0 \quad x = 4 \quad x = -4}$

34) $4x^3 - 8x^2 - 25x + 50$

$4x^2(x-2) - 25(x-2)$

$(4x^2 - 25)(x-2)$

$(2x-5)(2x+5)(x-2) = 0$

$\boxed{x = \frac{5}{2} \quad x = -\frac{5}{2} \quad x = 2}$

35) $8x^3 + 27 = (2x)^3 + (3)^3$

$= \boxed{(2x+3)(4x^2 - 6x + 9)}$

Rules:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

39) $(2x+3)(4x^2-6x+9)=0$

$2x+3=0$ $\qquad$ $4x^2-6x+9=0$

$2x=-3$ $\qquad$ No Real solutions

$\boxed{x=-\frac{3}{2}}$

40) $3\sin^2 x = \cos^2 x$

$3\sin^2 x = 1-\sin^2 x$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -\frac{1}{2}$

$$\boxed{\begin{array}{ll} x=\frac{\pi}{6} & x=\frac{7\pi}{6} \\ x=\frac{5\pi}{6} & x=\frac{11\pi}{6} \end{array}}$$

41) $\cos^2 x - \sin^2 x = \sin x$

$1-\sin^2 x - \sin^2 x = \sin x$

$1-2\sin^2 x = \sin x$

$0 = 2\sin^2 x + \sin x - 1$

$0 = (2\sin x - 1)(\sin x + 1)$

$2\sin x - 1 = 0$ $\qquad$ $\sin x + 1 = 0$

$2\sin x = 1$ $\qquad$ $\sin x = -1$

$\sin x = \frac{1}{2}$

$$\boxed{x=\frac{\pi}{6} \text{ or } \frac{5\pi}{6} \qquad x=-\frac{\pi}{2}}$$

42) $\tan x + \sec x = 2\cos x$

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$ $\quad > \cos x \neq 0$

$\sin x + 1 = 2\cos^2 x$

$\sin x + 1 = 2(1-\sin^2 x)$

$\sin x + 1 = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

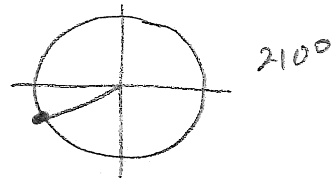
$(2\sin x - 1)(\sin x + 1) = 0$

Note! b/c $\cos x \neq 0$

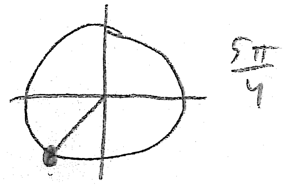
$x \neq -\frac{\pi}{2}$

From 41) $\boxed{\begin{array}{ll} x=\frac{\pi}{6}+2k\pi & \text{for all integer} \\ x=\frac{5\pi}{6}+2k\pi & \text{values of } k \end{array}}$

43) $\cos 210° = \boxed{-\frac{\sqrt{3}}{2}}$

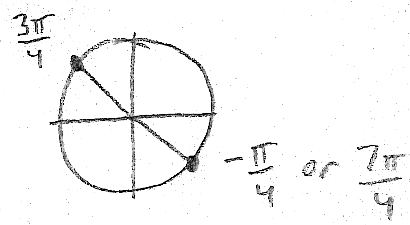


44) $\sin \frac{5\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$



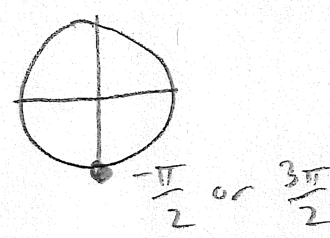
45) $\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

$\left(\text{or } \frac{3\pi}{4}, \text{or } \frac{7\pi}{4}\right)$

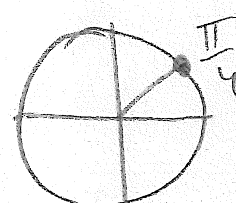


46) $\sin^{-1}(-1) = \boxed{-\frac{\pi}{2}}$

$\left(\text{or } \frac{3\pi}{2}\right)$

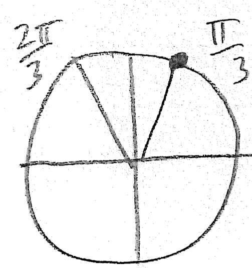


17) $\cos \frac{9\pi}{4} = \cos \frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$



48) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$

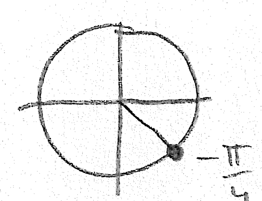
$\left(\text{or } \frac{2\pi}{3}\right)$



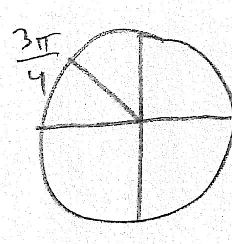
9) $\cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

$\sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$



$\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$



51-55) see other answers

56) $4x^2+12x+3=0$

$$x=\frac{-12\pm\sqrt{(12)^2-4(4)(3)}}{2(4)}$$

$$x=\frac{-12\pm\sqrt{144-48}}{8}$$

$$x=\frac{-12\pm\sqrt{96}}{8}$$

$$x=\frac{-12}{8}\pm\frac{4\sqrt{6}}{8}$$

$$\boxed{x=\frac{-3\pm\sqrt{6}}{2}}$$

57) $(x+2)(2x+1)=\left(\frac{5}{x+2}\right)(x+2)$

$2x^2+x+4x+2=5$

$2x^2+5x+2=5$

$2x^2+5x-3=0$

$\frac{-6}{6 \quad -1}$

$2x^2+6x-x-3=0$

$2x(x+3)-1(x+3)=0$

$(2x-1)(x+3)=0$

$\boxed{x=\frac{1}{2} \text{ or } x=-3}$

58) $\frac{x+1}{x}-\frac{x}{x+1}=0$

$$\frac{(x+1)(x+1)}{x(x+1)}-\frac{(x)(x)}{x(x+1)}=0$$

$$\frac{x^2+2x+1-x^2}{x(x+1)}=0$$

$$\frac{2x+1}{x(x+1)}=0$$

$2x+1=0$

$2x=-1$

$\boxed{x=-\frac{1}{2}}$

59)

$$x^4-6x^3+13x^2-26x+45+\frac{-89}{x+2}$$

$$
\begin{array}{r|l}
x+2 & x^5-4x^4+x^3+0x^2-7x+1 \\
 & -x^5 \mp 2x^4 \\
\hline
 & -6x^4+x^3 \\
 & +6x^4+12x^3 \\
\hline
 & 13x^3+0x^2 \\
 & -13x^3 \mp 26x^2 \\
\hline
 & -26x^2-7x \\
 & +26x^2+52x \\
\hline
 & 45x+1 \\
 & -45x \mp 90 \\
\hline
 & -89
\end{array}
$$

60) $x^3+0x^2+0x+1 \,\overline{\big)\, x^5 - x^4 + x^3 + 2x^2 - x + 4}$

$$x^2 - x + 1 + \frac{x^2+3}{x^3+1}$$

$$\cancel{x^5} - x^4 + x^3 + 2x^2 - x + 4$$
$$-\cancel{x^5} \mp 0 \mp 0 + {}^-x^2$$
$$-x^4 + x^3 + x^2 - x$$
$$+x^4 \mp 0 \mp 0 + x$$
$$\cancel{x^3} + x^2 + 0x + 4$$
$$-\cancel{x^3} \mp 0 \mp 0 \mp 1$$
$$x^2 + 3$$

61)

2	12	-23	-3	2
		24	2	-2
	12	1	-1	$\boxed{0}$

$12x^2 + 1x - 1 = 0$

$12x^2 + 4x - 3x - 1 = 0$

$4x(3x+1) - 1(3x+1) = 0$

$(4x-1)(3x+1) = 0$

-12

$4 \quad -3$

$4x - 1 = 0 \qquad 3x + 1 = 0$

$4x = 1 \qquad 3x = -1$

$\boxed{x = \frac{1}{4}} \qquad \boxed{x = -\frac{1}{3}}$

62)

$$\begin{array}{r|rrrr} -\frac{1}{2} & 12 & 8 & -1 & -1 \\ & & -6 & -1 & 1 \\ \hline & 12 & 2 & -2 & \boxed{0} \end{array}$$

Tested b/c rational roots test

worked

$\boxed{x = -\frac{1}{2} \text{ is a solution}}$

$$12x^2 + 2x - 2 = 0$$

$$6x^2 + x - 1 = 0$$

$$6x^2 + 3x - 2x - 1 = 0$$

$$3x(2x+1) - 1(2x+1) = 0$$

$$(3x-1)(2x+1) = 0$$

-6

$3 \quad -2$

$3x - 1 = 0 \qquad 2x + 1 = 0$

$3x = 1 \qquad 2x = -1$

$\boxed{x = \frac{1}{3}} \qquad \boxed{x = -\frac{1}{2}}$

---

63) $x^2 + 2x - 3 \leq 0$

$(x+3)(x-1)$

$x = -3 \qquad x = 1$

		-3		1	
Test	$f(-4)$		$f(0)$		$f(2)$
Sign	$+$		$-$		$+$

$\boxed{[-3, 1]}$

64) Find Critical values

$$\frac{2x-1}{3x-2} = 1$$

$2x - 1 = 3x - 2$

$1 = x$

note:

$3x - 2 \neq 0$

$3x \neq 2$

$x \neq \frac{2}{3}$

	$\frac{2}{3}$		1	
$f(0) < 1$		$f(\frac{5}{6}) > 1$		$f(2) < 1$
✓		X		✓

$$\left(-\infty, \frac{2}{3}\right) \cup [1, \infty)$$

65) Critical Values: $2x+3=0$

$2x=-3$

$x=\frac{-3}{2}$

$x-5=0$

$x=5$

$\frac{2}{2x+3} = \frac{2}{x-5}$

$2x-10 = 4x+6$

$-16 = 2x$

$-8 = x$

$-8 \qquad -\frac{3}{2} \qquad 5$

$f(-10)$ ✓ $\quad f(-4)$ X $\quad f(0)$ ✓ $\quad f(6)$ X

Test in original inequality

$\boxed{(-\infty, -8) \cup (-\frac{3}{5}, 5)}$

66) $-x+4 \leq 1$

$-x \leq -3$

$x \geq 3$

$-x+4 \geq -1$

$-x \geq -5$

$x \leq 5$

$\boxed{[3, 5]}$

67) $5x-2=8$

$5x=10$

$\boxed{x=2}$

$5x-2=-8$

$5x=-6$

$\boxed{x=\frac{-6}{5}}$

68) $2x+1>3$

$2x>2$

$x>1$

$2x+1<-3$

$2x<-4$

$x<-2$

$\boxed{(-\infty, -2) \cup (1, \infty)}$

69) $m = \frac{-4-3}{2-1} = \frac{-7}{3}$

Point-slope $\quad y-3 = -\frac{7}{3}(x - -1)$

$3y-9 = -7x-7$

Standard $\boxed{7x+3y=2}$

70) Find slope: $2x-3y+5=0$

$-3y = -2x-5$

$y = \boxed{\frac{2}{3}}x + \frac{5}{3}$

$m_{\perp} = -\frac{3}{2}$

$y-2 = -\frac{3}{2}(x - -1)$

$2y-4 = -3x-3$

$\boxed{3x+2y=1}$

71) Midpoint: $\left(\frac{-1+3}{2}, \frac{4+2}{2}\right) \to (1,3)$

$m = \frac{3-3}{1-2} = 0$

$y - 3 = 0(x-2)$

$\boxed{y = 3}$

72) $5(3x - y - 7 = 0)$
$x + 5y + 3 = 0$

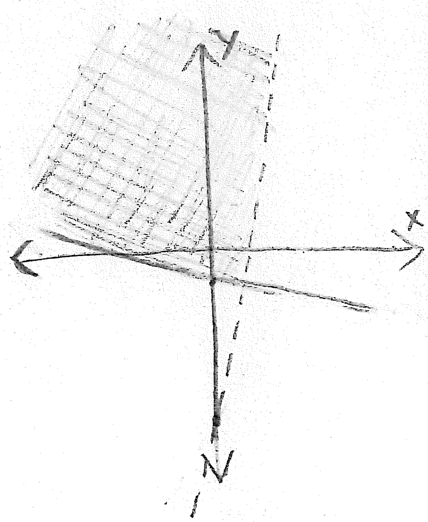
$15x - \cancel{5y} - 35 = 0$
$x + \cancel{5y} + 3 = 0$
$16x - 32 = 0$
$16x = 32$
$x = 2$

Plug in
$2 + 5y + 3 = 0$
$5y = -5$
$y = -1$
$\boxed{(2,-1)}$

73) Get into slope-intercept

$y > 3x - 7$

$y \geq \frac{-1}{5}x - \frac{3}{5}$

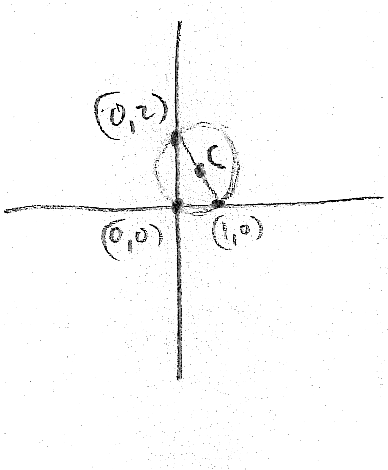


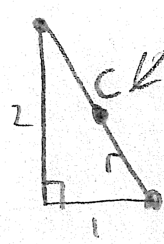
74) $(x-1)^2 + (y-2)^2 = r^2$
$= 18$

radius is the distance between the points.

$\sqrt{3^2 + 3^2}$
$= \sqrt{18}$

75)





Center of circle

r = radius of circle

$r = \frac{1}{2}\sqrt{1^2 + 2^2}$

$r = \frac{1}{2}\sqrt{5}$

$r = \frac{\sqrt{5}}{2}$

Center = Midpoint

$\left(\frac{1+0}{2}, \frac{0+2}{2}\right)$

$\left(\frac{1}{2}, 1\right)$

$\boxed{\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = \left(\frac{\sqrt{5}}{2}\right)^2 = \frac{5}{4}}$

76) Rearrange and complete the square

$$x^2+y^2+6x-4y+3=0$$

$$x^2+6x+\underbrace{9}+y^2-4y+\underbrace{4} = -3+\underbrace{9}+\underbrace{4}$$

$$(x+3)^2+(y-2)^2=10$$

$$\boxed{\text{Center: } (-3,2) \quad \text{radius} = \sqrt{10}}$$

77) $\sqrt{x^2+x-2} \rightarrow \quad x^2+x-2>0$

Note: Can't equal "0" b/c in Denominator.

Can't take sq. root of negative #.

$(x-1)(x+2)$

$x=1 \quad x=-2$

$-2 \quad 1$

test: $f(-3) \quad f(0) \quad f(2)$

Sign: $+ \quad - \quad +$

$$\boxed{\text{Domain: } (-\infty,-2)\cup(1,\infty)}$$

78) D: $(-\infty,\infty)$ R: $\{7\}$	79) D: $(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},\infty)$ R: $(-\infty,\frac{5}{2})\cup(\frac{5}{2},\infty)$	80) D: $(-\infty,0)\cup(0,\infty)$ R: $\{-1,1\}$

81) $\dfrac{2(x+h)+3-(2x+3)}{h} = \dfrac{2x+2h+3-2x-3}{h} = \dfrac{2h}{h} = \boxed{2}$

82) $$\frac{\left(\frac{1}{(x+h)+1} - \frac{1}{x+1}\right)\left((x+h+1)(x+1)\right)}{(h)\quad\left((x+h+1)(x+1)\right)}$$

$$= \frac{x+1-(x+h+1)}{h(x+h+1)(x+1)} = \frac{\cancel{x}\cancel{+1}\cancel{-x}-h\cancel{-1}}{h(x+h+1)(x+1)} = \frac{-h}{h(x+h+1)(x+1)}$$

$$= \boxed{\frac{-1}{(x+h+1)(x+1)}}$$

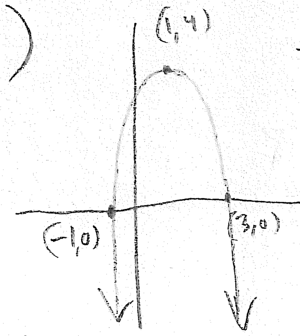
83)
$$\frac{3(x+h)^2-(x+h)+5 \;-(3x^2-x+5)}{h}$$

$$= \frac{\cancel{3x^2}+6xh+3h^2\cancel{-x}-h\cancel{+5}\cancel{-3x^2}\cancel{+x}\cancel{-5}}{h}$$

$$= \frac{6xh+3h^2-h}{h} = \frac{\cancel{h}(6x+3h-1)}{\cancel{h}} = \boxed{6x+3h-1}$$

84 – 90: See other Answers

89)



$-(x+1)(x-3)=y$

$\boxed{-x^2+2x+3=y}$

91) $f(x)=2x+3$

$y=2x+3$

$x=2y+3$

$x-3=2y$

$\frac{x-3}{2}=y$

$\boxed{\frac{x-3}{2}=f^{-1}(x)}$

92) $y=\frac{x+2}{5x-1}$

$x=\frac{y+2}{5y-1}$

$5yx-x=y+2$

$5yx-y=x+2$

$y(5x-1)=x+2$

$y=\frac{x+2}{5x-1}$

$\boxed{f^{-1}(x)=\frac{x+2}{5x-1}}$

93) $y = x^2 - 2x - 1$

$x = y^2 - 2y - 1$

$x + 1 = y^2 - 2y$

$x + 1 + 1 = y^2 - 2y + 1$

$x + 2 = (y-1)^2$

$\sqrt{x+2} = y - 1$

$\sqrt{x+2} + 1 = y$

$\boxed{f^{-1}(x) = \sqrt{x+2} + 1}$

94) See other answers

95) $\dfrac{r}{x+t} = \dfrac{h}{t}$

$tr = hx + ht$

$tr - ht = hx$

$\dfrac{tr - ht}{h} = x$

$\boxed{t\left(\dfrac{r-h}{h}\right) = x}$

97) $\dfrac{(2r)^2 - \pi r^2}{(2r)^2}$

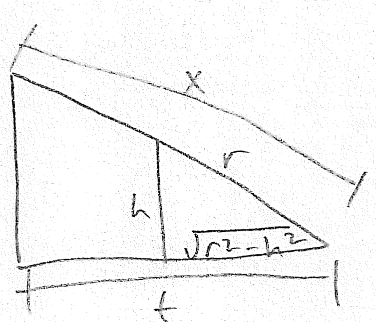
$= \dfrac{4r^2 - \pi r^2}{4r^2}$

$= \boxed{\dfrac{4-\pi}{4}}$

98) $\frac{1}{2}(2\pi r) + r + 2r + r$

$\boxed{\pi r + 4r}$

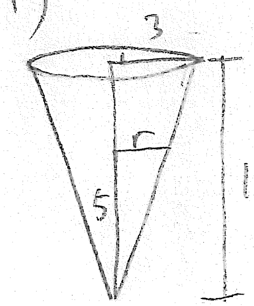
96)



$\dfrac{x}{t} = \dfrac{r}{\sqrt{r^2-h^2}}$

$\boxed{x = \dfrac{rt}{\sqrt{r^2-h^2}}}$

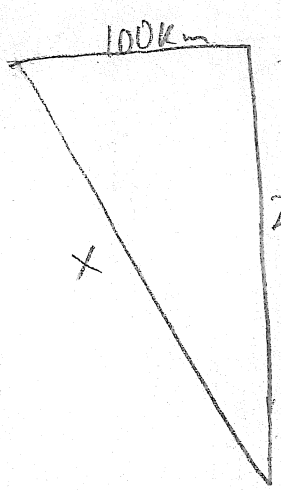
99)



$\dfrac{3}{10} = \dfrac{r}{5} \rightarrow r = \dfrac{3}{2}$

$A = \left(\dfrac{3}{2}\right)^2 \pi = \boxed{\dfrac{9}{4}\pi}$

100)



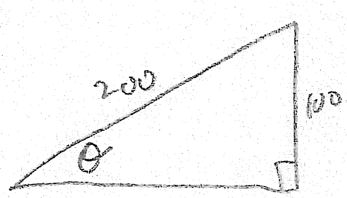
$x = \sqrt{100^2 + 200^2}$

$x = \sqrt{50000}$

$= \sqrt{10000} \cdot \sqrt{5}$

$= \boxed{100\sqrt{5}\text{ km}}$

101)



$\sin\theta = \dfrac{100}{200}$

$\sin\theta = \dfrac{1}{2}$

$\boxed{\theta = \dfrac{\pi}{6}}$

102) $f(g(x)) = \boxed{2\sqrt{3x-1} - 3}$

103) $g(f(x)) = \sqrt{3(2x-3)-1}$

$= \sqrt{6x-9-1}$

$= \boxed{\sqrt{6x-10}}$

104) $f(g(x)) = \dfrac{3}{\left(\frac{x}{2x-1}\right)}$

$= \dfrac{3(2x-1)}{x}$

$= \dfrac{6x-3}{x}$ $\quad x \neq 0$

$x \neq \frac{1}{2}$

D: $(-\infty, 0) \cup (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

105 - 108: See other answers